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Magnetooscillation of photocurrent in mesoscopic metal films

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Abstract
The oscillation of a photocurrent produced by illumination of a mesoscopic metal film as a function of external magnetic field is considered. The orientation of the magnetic field **B** is assumed to be arbitrary. The thickness of the film is supposed to be much larger than the penetration depth of light but much smaller than the electron mean free path due to the scattering in the bulk of the sample. The oscillation of the current in the plane of light incidence is analyzed.

Our theory predicts oscillations of two types. The first type can exist for any constant energy surface, including spheres. The second type can exist only for nonspherical constant energy surfaces. The amplitudes of the oscillations of the first and second type are proportional, respectively, to B^{-3} and $B^{-3/2}$.

1. Introduction

The purpose of the present paper is the theoretical investigation of an oscillatory dependence of the photocurrent in mesoscopic metal films as a function of tilted external magnetic field **B**. Previously, in the papers by Gurevich *et al* [1] and Gurevich and Laiho [2], it was shown experimentally and theoretically that light falling obliquely on a plane surface of a normal metal sample produces a surface photocurrent. This phenomenon has some similarity to the drag of electrons caused by traveling electromagnetic waves in semiconductors which was first considered by Barlow [3] (see also [2] and the references therein). It is closely related to the photogalvanic effect in semiconductors—see the papers by Magarill and Entin [4] and Alperovich *et al* [5]. In these papers, as well as in the present one, the considered effect is associated with diffuse scattering of electrons from the surface(s) of the sample.

The experimental paper by Böhm *et al* [6], in which the magnetooscillation under illumination of tungsten single crystals was investigated, has motivated the present theory. In that paper the influence of a strong magnetic field, in combination with nonhomogeneous illumination of the sample, on ballistic carrier transport was studied.

The most important difference between effects in semiconductors and metals is in the conditions for observation of this phenomenon. Because of a comparatively small conductivity of semiconductors, one usually measures the voltage built up across the sample. In metals, because of their high conductance, only direct measurement of the photocurrent is usually possible as the voltage is quite small. Besides, the physical situation in metals is extremely nonhomogeneous as the light penetration depth (κ^{-1}) is usually about 10^{-6} cm. This length for a number of cases of interest is much smaller than both the thickness of the film a and the electron mean free path l while the situation in semiconductors (at least regarding the electron mean free path) is usually the opposite. The term mesoscopic is widely used to describe physical effects that are sensitive to dimensions of the sample. In the present paper we will assume that the thickness of a sample is smaller than the electron mean free path. Such samples we will call mesoscopic.

An oscillation of the photocurrent in a mesoscopic metal film as a function of applied magnetic field for a special geometry and the simplest assumptions about the electron dispersion law are investigated theoretically in [7]. Only the geometry where magnetic field **B** is perpendicular to the film surface has been considered. This assumption has been fundamentally essential in working out the theory as, in this case, the plane where the electron motion is finite coincides with the plane of the film.

However, the experimental setup usually demands a nonperpendicular field orientation. This poses an entirely new problem for the theory as the plane where the motion of electrons becomes finite does not coincide with the plane of the film. In addition, it is desirable to calculate the period of oscillation for an arbitrary spectrum and find out whether new types of oscillation can exist for more complicated electron dispersion laws. These points mean that a new mathematical apparatus should be developed to solve the problem. To treat the case where the magnetic field is tilted, as well as the electron spectrum being of an arbitrary form, one should solve a partial derivative differential equation. This problem is much more involved.

An appropriate approach has been developed in the paper by Gurevich [8] who considered the magnetooscillation of conductivity of Sondheimer [9] type in a metal for an arbitrary electron spectrum and magnetic field direction. (See also the book by Abrikosov [10], section 8.5 where a very good and detailed review of these effects is given.) However, only the spatially homogeneous case is treated in these papers. Therefore one has to modify this approach for a nonhomogeneous situation characteristic of the photocurrent in metals.

In the present paper we will consider an arbitrary orientation of magnetic field in the plane of light incidence and show that the theory with an arbitrary electron spectrum predicts oscillation of two types. The first type can be called the limiting point oscillation as it is associated with a limiting point of the constant energy surface for a given direction of magnetic field **B**. A limiting point is determined by the maximal value of the electron quasimomentum in the direction of magnetic field. Near such a point the electron trajectories in the plane of the plate have small dimensions. As a result, such an oscillation dies out rather quickly with magnetic field B. This type of oscillation can exist for any sort of closed constant energy surface, including spheres.

The second type of oscillation does not exist for spheres but can exist for more complicated sorts of constant energy surfaces. It is associated with the situation where the helical pitch of the electron trajectory in a magnetic field passes through an extremum. In this case the electrons have comparatively large trajectories in the plane of the plate. The authors of [6] claim that the oscillations they have observed belong to this type.

The amplitude of the oscillation of the first kind is proportional for high magnetic fields B to B^{-3} while the amplitude of the oscillation of the second kind is proportional to $B^{-3/2}$ for the effect treated in the present paper. So inhomogeneity of the perturbation increases by 1 the powers of magnetic field in the asymptotic (where the oscillation is periodic) as compared with oscillation of Sondheimer type [8, 9]. Naturally, the period of the oscillation depends on the angle χ between the magnetic field **B** and the perpendicular to the surface of the film.

2. Nonequilibrium distribution function in valence band calculations of photocurrent

2.1. Boltzmann equation

As mentioned in section 1 we consider the problem that is similar to the one treated in [7] but for the case where magnetic field is tilted and the electron spectrum is more complicated. Accordingly, we should reformulate the Boltzmann equation

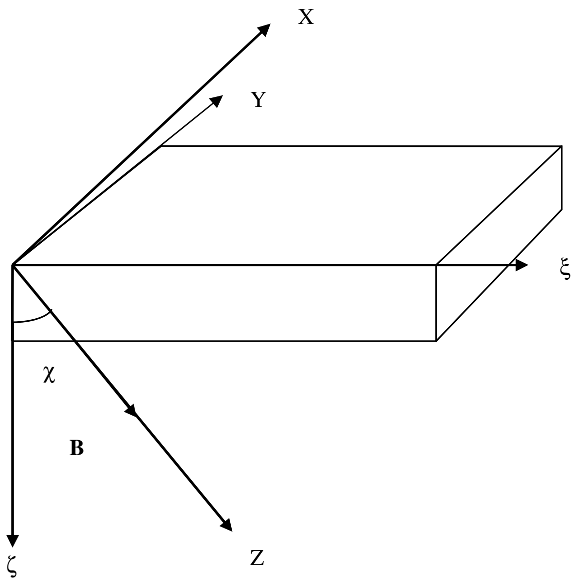

Figure 1. Rectangular coordinate systems (x, y, z) and (ξ, η, ζ). The z-axis is parallel to **B** while x- and y-axes are perpendicular to it. The ζ-axis is perpendicular to the plane of the film. It makes angle χ with the z-axis. ξ- and x-axes are in the plane of propagation of the light beams. y- and η-axes coincide.

for this case. We shall not discuss the limits of its applicability, referring the reader instead to the paper [7] where they are discussed in detail.

Below we will use the following rectangular coordinate systems—see figure 1. The z-axis is parallel to **B** while the x-and y-axes are perpendicular to it. The ζ-axis is assumed to be perpendicular to the plane of the film. It makes angle χ with the z-axis. The ξ- and x-axes are in the plane of propagation of the light beams; the y- and η-axes coincide. In the present paper the magnetic field dependence of the photoinduced current density $\overline{j}_\xi$ is investigated. Here $\overline{j}_\xi$ is the ξ-component of the dc current density averaged over the film cross section.

The oscillation we consider is due to diffuse scattering of the electrons from the surfaces of the film. Their distribution function is a nonequilibrium one due to the interband transitions from the lower band (1) to the upper one (2) brought about by the ac electric field of light **E**. One can easily check that the transition probability of an electron is invariant to the change of sign of the electron quasimomentum. Therefore the number of electrons generated by light just near the surface $\zeta = 0$ and moving from the surface and to the surface are the same. If the scattering is diffuse and the penetration depth of light is much smaller than the sample thickness a the electrons moving from the surface $\zeta = a$ give no contribution to the oscillation as this surface is not illuminated. Due to the electrons moving from the surface $\zeta = 0$ the oscillation of $\overline{j}_\xi$ as a function of external field **B** takes place. This is why we will consider only the electrons moving from the surface $\zeta = 0$ illuminated by light. (If the electron reflection from the surfaces were specular, current oscillation of this type would not exist because of invariance of the transition probability to the change of sign of the electron quasimomentum.)

The Boltzmann equation for this case can be written in the form

$$v_\zeta \frac{\partial f_{\mathbf{p}}}{\partial \zeta} + \Omega \frac{\partial f_{\mathbf{p}}}{\partial \tau} + \left[\frac{\partial f_{\mathbf{p}}}{\partial t}\right]_{\text{coll}} = d \exp(-\kappa\zeta), \tag{1}$$

where the subscript 'coll' indicates the collision term and d is given by

$$d = \frac{\pi}{2\hbar}\left(\frac{e}{m_0\omega}\right)^2 |\mathbf{E}_0\mathbf{P}_{21}(\mathbf{p},\mathbf{p}')|^2 \delta(\epsilon^{(1)} + \hbar\omega - \epsilon^{(2)}). \tag{2}$$

Here $f_{\mathbf{p}}$ is the electron distribution function, $\mathbf{E}_0$ is the amplitude of the electric field, e is the electron charge, ω is the frequency of light, m_0 is the free electron mass, $\mathbf{P}_{21}(\mathbf{p},\mathbf{p}')$ is the interband transition amplitude, and

$$\Omega = \frac{eB}{m^* c} \tag{3}$$

is the cyclotron frequency of an electron in a magnetic field,

$$m^* = \frac{1}{2\pi}\frac{\partial S}{\partial \epsilon} \tag{4}$$

is the electron cyclotron effective mass on its trajectory in a magnetic field; these quantities depend on the electron energy ϵ and p_z. S is the area of the cross section of the surface $\epsilon(\mathbf{p}) = \text{const}$ by the plane $p_z = \text{const}$, τ is a dimensionless time of motion of an electron on its trajectory in a magnetic field (see for instance the book by Abrikosov [10], section 5.1) and $\mathbf{v}$ is the electron velocity on the trajectory. We assume that the characteristic scale of the electromagnetic wave variation is bigger than the scale of variation of the electron wavefunction. This means that in the matrix element

$$\int \mathrm{d}^3 p \Psi_1(\mathbf{p}-\mathbf{k}/2)\Psi_2^*(\mathbf{p}+\mathbf{k}/2)\,(\mathbf{p}+\mathbf{k}/2, \mathbf{E}(\mathbf{k}))$$

one can neglect the small item $\mathbf{k}$ in comparison with $\mathbf{p}$. (The last term in brackets is the scalar product of two vectors $\mathbf{p}+\mathbf{k}/2$ and $\mathbf{E}$; $\Psi_{1(2)}$ are the electron wavefunctions in the lower (upper) band.) This permits one to rewrite the exact matrix element in the form proportional to

$$|\mathbf{E}_0\mathbf{P}_{21}(\mathbf{p},\mathbf{p}')|^2 \exp(-\kappa\zeta), \tag{5}$$

with

$$\mathbf{P}_{21}(\mathbf{p},\mathbf{p}') = -\mathrm{i}\hbar \int \mathrm{d}^3 r u_{\mathbf{p}}^{(2)*}(r)\nabla u_{\mathbf{p}'}^{(1)}(r) \tag{6}$$

where $p_\xi = p'_\xi + \hbar k_\xi$ and $u^{(1,2)}$ are the Bloch amplitudes of the electron wavefunctions.

2.2. Isotropic spectrum

To begin with, we will discuss the simplest model with an isotropic quadratic electron spectrum:

$$\epsilon^{(1)}(p) = p^2/2m_1 \qquad \epsilon^{(2)}(p) = \epsilon_{\mathrm{g}} + p^2/2m_2. \tag{7}$$

Here the mass m_1 and gap ϵ_{g} can be both negative and positive. Further on we will assume that

$$|m_1| \gg m_2.$$

Therefore we will consider contributions to the current only from the electrons of the upper (conductance) band 2.

To carry out the integration over quasimomenta, we write the argument of the δ-function in equation (2) as $p^2 - p_{\omega'}^2$ where

$$p_{\omega'}^2 = 2M\left(\hbar\omega - \epsilon_{\mathrm{g}}\right)$$

and $M^{-1} = m_1^{-1} + m_2^{-1}$.

In order to solve equation (1) one should take into account that the quasimomenta p_ξ and p_ζ are functions of the dimensionless trajectory time τ:

$$\begin{aligned} p_\zeta &= p_z \cos\chi - p_\perp \cos\tau \sin\chi \\ p_\xi &= p_z \sin\chi + p_\perp \cos\tau \cos\chi. \end{aligned} \tag{8}$$

So, the right-hand side of the Boltzmann equation, $d(\tau)$, is a function of the 'trajectory time' τ.

The distribution function should obey the diffuse reflection boundary conditions at both surfaces of the film

$$\begin{aligned} f &= 0 \qquad \text{for } \zeta = 0, \quad v_\zeta > 0, \\ f &= 0 \qquad \text{for } \zeta = a, \quad v_\zeta < 0. \end{aligned} \tag{9}$$

To solve equation (1) we will expand $f(\zeta,\tau)$ as a function of ζ into a Fourier series

$$f(\zeta,\tau) = a^{-1}\sum_n \phi_n(\tau)\exp(2\pi \mathrm{i} n\zeta/a),$$

so that

$$\frac{\partial\phi_n(\tau)}{\partial\tau} + \frac{2\pi n v_\zeta}{\Omega a}\phi_n(\tau) + \frac{|v_\zeta|}{\Omega}[f_{\mathbf{p}}(0,\tau)\theta(-v_\zeta) + f_{\mathbf{p}}(a,\tau)\theta(v_\zeta)] + \gamma\phi_n = d_n(\tau). \tag{10}$$

Here $\gamma = 1/\Omega t_0$ is a dimensionless reciprocal collision time, while t_0 is the usual collision time and d_n is the Fourier component of $d(\zeta,\tau)$. One can write the solution of this equation for $v_\zeta > 0$ at $z = a$ as

$$\phi_n(\tau) = \frac{1}{\Omega}\int_{-\infty}^{\tau}\mathrm{d}\tau'\Bigg[d_n(\tau') - v_\zeta f_{\mathbf{p}}(a,\tau') \times \exp\left(\int_\tau^{\tau'}\mathrm{d}\tau''\left(\mathrm{i}\frac{2\pi n}{\Omega a}v_\zeta + \gamma\right)\right)\Bigg]. \tag{11}$$

The function $f_{\mathbf{p}}(a,\tau')$ on the right-hand side of the equation should satisfy the self-consistency condition

$$\frac{1}{2}f_{\mathbf{p}}(a,\tau) = \frac{1}{a\Omega}\int_{-\infty}^{\tau}\mathrm{d}\tau'\sum_n\Bigg[d_n(\tau') \times \exp\left(\int_\tau^{\tau'}\mathrm{d}\tau''\left(\mathrm{i}\frac{2\pi n}{\Omega a}v_\zeta + \gamma\right)\right) - v_\zeta f_{\mathbf{p}}(a,\tau')\exp\left(\int_\tau^{\tau'}\mathrm{d}\tau''\left(\mathrm{i}\frac{2\pi n}{\Omega a}v_\zeta + \gamma\right)\right)\Bigg]. \tag{12}$$

Making use of the well-known identity

$$\sum_k \exp(\mathrm{i}kx) = 2\pi\sum_n \delta(x - 2\pi n), \tag{13}$$

one can rewrite equation (12) in the form

$$\frac{1}{2}f_{\mathbf{p}}(a,\tau) = G_{\mathbf{p}}(\tau) - \frac{2\pi}{\Omega a}\int_{-\infty}^{\tau}\mathrm{d}\tau' v_\zeta(\tau') f_{\mathbf{p}}(a,\tau') \times \sum_m \delta\left(\frac{2\pi}{\Omega a}\int_\tau^{\tau'}\mathrm{d}\tau'' v_\zeta(\tau'') - 2\pi m\right). \tag{14}$$

Here

$$G_{\mathbf{p}}(\tau) = \frac{1}{a\Omega}\sum_n \int_{-\infty}^{\tau} d\tau'\, d_{\xi n}(\tau') \times \exp\left(\int_{\tau}^{\tau'} d\tau''\left(i\frac{2\pi n}{\Omega a}v_\zeta(\tau'') + \gamma\right)\right). \tag{15}$$

The solution of this equation is

$$f_{\mathbf{p}}(a,\tau) = G_{\mathbf{p}}(\tau) - G_{\mathbf{p}}[\tau_1(\tau)]\exp\left[-\gamma(\tau-\tau_1)\right], \tag{16}$$

where $\tau_1(\tau)$ is the root of equation

$$\frac{1}{\Omega a}\int_{\tau_m}^{\tau} d\tau' v_\zeta(\tau') = m \tag{17}$$

for $m = 1$. Here, to verify that equation (16) is the solution of equation (15) we have made use of the identity

$$\tau_n\left(\tau_m(\tau)\right) = \tau_{n+m}(\tau).$$

By virtue of equation (8) one can write equation (17) in the form

$$\tau_1 - \frac{p_\perp}{p_z}\tan\chi\sin\tau_1 = -\frac{\Omega a m_2}{p_z\cos\chi} + \tau - \frac{p_\perp}{p_z}\tan\chi\sin\tau. \tag{18}$$

This transcendental equation can be solved for small magnetic fields ($\Omega a m_2/p_{\omega'} \ll 1$) as well as for large ones ($\Omega a m_2/p_{\omega'} \gg 1$). The first situation is not interesting as there is no oscillation in this region of magnetic fields. For the second case one has

$$\tau_1 = \tau - \frac{\Omega a m_2}{p_z\cos\chi}. \tag{19}$$

We also assume that the angle $\vartheta = \pi/2 - \chi$ is sufficiently large, namely

$$\vartheta \gg p_{\omega'}/\Omega a m_2.$$

Otherwise equation (18) has no solution.

As we treat a spatially nonhomogeneous perturbation of the electron system, the oscillating part of the distribution function can be written in the form

$$\phi_n^{\text{osc}}(\tau) = -\frac{1}{\Omega\kappa}\exp\left(-\gamma\frac{\Omega a m_2}{p_z\cos\chi}\right)\int_{-\infty}^{\tau} d\tau'\, d(\tau')\delta(\epsilon^{(1)} + \hbar\omega - \epsilon^{(2)})\exp\left[\int_{\tau}^{\tau'} d\tau''\left(i\frac{2\pi n}{\Omega a}v_\zeta + \gamma\right)\right]. \tag{20}$$

We are interested in the oscillating part of the current density averaged over the width of the film

$$\bar{j}_\xi = \frac{1}{a}\int_0^a j_\xi\, d\xi, \tag{21}$$

i.e. one needs only the Fourier component with $n = 0$. The matrix element $d(\tau')$ entering equation (20) depends on the amplitudes of the interband transitions $\mathbf{P}_{21}(\mathbf{p},\mathbf{p}')$. Later, we will accept the simplest form

$$\mathbf{P}_{21}(\mathbf{p},\mathbf{p}') = \alpha(\mathbf{p}+\mathbf{p}') \approx 2\alpha\mathbf{p} \tag{22}$$

where α is a dimensionless constant. Such a form of $\mathbf{P}_{21}(\mathbf{p})$ is obvious for an isotropic model and gives a correct estimation for the magnitude of the effect for a more complicated electron spectrum. Neither the period of oscillation given by the asymptotic of the function $\bar{j}_\xi$ for large values of B nor the law of decrease of the oscillation amplitude with B depend on this assumption.

The expressions like $\mathrm{Re}(E_{\zeta 0}E^*_{\xi 0})$ denote the time average of two amplitudes. One can see below that for the calculation of the high magnetic field asymptotic we are interested in we can omit in $|\mathbf{E}_0\mathbf{p}|^2$ all the terms with the exception of $2\,\mathrm{Re}(E_{\zeta 0}E^*_{\xi 0})p_\xi p_\zeta$.

So, the oscillating part of the contribution to the current can be written in the form

$$\bar{j}_\xi = \frac{2m_2 e}{(2\pi\hbar)^3}\int_0^{p_{\omega'}} dp_z \int_0^{2\pi} d\tau\, v_\xi \phi_0^{\text{osc}}(\tau). \tag{23}$$

As a result one has

$$\bar{j}_\xi = j_0\Phi(\chi, B). \tag{24}$$

Here

$$j_0 = \frac{e\alpha^2 p_{\omega'}^4}{\pi\hbar^4\Omega\kappa a}\left(\frac{e}{m_0\omega}\right)^2 \mathrm{Re}\left(E_{0\zeta}E^*_{0\xi}\right) \tag{25}$$

and the function $\Phi(\chi, B)$ of an angle χ describes an oscillatory dependence of the current on the magnitude and direction of the magnetic field

$$\Phi(\chi,B) = \frac{1}{p_{\omega'}^4}\int_0^{p_{\omega'}} dp_z\, p_z\left(p_{\omega'}^2 - p_z^2\right)\sin\left(\frac{\Omega a m_2}{p_z\cos\chi}\right) \sim 2\left(\frac{p_{\omega'}\cos\chi}{\Omega a m_2}\right)^2 \sin\left(\frac{\Omega a m_2}{p_{\omega'}\cos\chi}\right)\cos\chi\cos 2\chi. \tag{26}$$

One can see that the period of oscillation is

$$\Delta B = \frac{2\pi c p_{\omega'}\cos\chi}{ea}, \tag{27}$$

and we get for the asymptotic of the oscillation in magnetic field

$$\bar{j}_\xi \propto \frac{1}{B^3}\sin\left(\frac{\Omega a m_2}{p_{\omega'}\cos\chi}\right). \tag{28}$$

It is assumed here that $B \gg B_a$ where

$$B_a = c p_{\omega'}/ea.$$

We see that for the magnetooscillation of the photocurrent we get a negative power of B smaller by 1 than for the ordinary Sondheimer effect [9]. For tilted magnetic field, the dependence of the period ΔB on $\cos\chi$ corresponds to an increase of the 'effective thickness' of the film according to

$$a \to a/\cos\chi.$$

2.3. Anisotropic spectrum

So far we have considered the simplest case of an isotropic electron spectrum in both bands. One can also treat the case of anisotropic spectrum where the lower band is so narrow that it can be approximated by a constant $\epsilon_{\mathbf{p}}^{(1)} = \epsilon_{\mathrm{g}}$. Then the conservation law in our approximation takes the form

$$\varepsilon_{\mathbf{p}}^{(2)} = \hbar\omega', \qquad \hbar\omega' = \hbar\omega - \epsilon_{\mathrm{g}}. \tag{29}$$

In the case where the constant energy surface (29) (we will call it the ω'-surface) in the direction of $\mathbf{B}$ is closed, one can use the treatment of [8] (see also [10]) to obtain the asymptotic form of the current oscillation. Assume that the plane perpendicular to $\mathbf{B}$ is tangential to the ω'-surface at the point $p_z^{(0)}$. Such a point as we indicated above is called a *limiting point*. Then using the results of [8], $1/p_{\omega'}$ should be replaced by $\sqrt{K}$, where K is the Gaussian curvature of the ω'-surface at the point $p_z^{(0)}$. As a result, one gets the following asymptotic for the ξ-component of the average current

$$\overline{j}_\xi \propto \frac{1}{B^3} \sin\left(\frac{aeB\sqrt{K}}{c\cos\chi}\right). \tag{30}$$

The period of the oscillation is

$$\Delta B = 2\pi c \cos\chi / ae\sqrt{K}. \tag{31}$$

Let us now consider oscillations of the second kind. We will assume now that the ω'-surface is closed but otherwise has an arbitrary form. For this case not only the first harmonic of $\overline{j}_\xi(B)$ in the oscillatory part of the average current exists but also the higher harmonics, in contrast with a quadratic electron spectrum. However, to calculate the period of oscillation it is sufficient to take into account only the first harmonic as the higher harmonics would not change the result for the principal period. Taking into account equation (26) one can rewrite the angular function $\Phi(\chi, B)$ in the form

$$\Phi(\chi, B) = \int_0^{p_{\omega'}} \mathrm{d}p_z\, L(p_z, \chi)\, \mathrm{Re} \times \left[\exp \mathrm{i}\left(\frac{2\pi aeB}{c\cos\chi\,(\partial S/\partial p_z)}\right)\right]. \tag{32}$$

where S is the area of the cross section of the ω'-surface formed by a plane $p_z = \text{const}$. Here we have made the following replacement in the argument of the sine that should be done for the ω'-surface of an arbitrary form

$$p_z \to \frac{1}{2\pi}\left(\frac{\partial S}{\partial p_z}\right) = m^*\overline{v}_z. \tag{33}$$

The function $L(p_z, \chi)$ is proportional to the matrix elements of electron–light interaction and is a smooth function of p_z and $\cos\chi$. Its exact form is immaterial for our purpose. We come to the conclusion that for an arbitrary value of p_z the distribution function is an oscillating function of B with period

$$\Delta B = c\cos\chi(\partial S/\partial p_z)/ae \tag{34}$$

that depends on the value p_z determining the cross section.

An oscillation of the second kind exists provided the quantity

$$\left(\frac{1}{2\pi}\frac{\partial S}{\partial p_z}\right)^{-1} = \frac{1}{m^*\overline{v}_z}$$

has an extremum at some value of $p_z = p_0$, so that in the vicinity of the extremum

$$\frac{1}{m^*\overline{v}_z} = \left(\frac{1}{m^*\overline{v}_z}\right)\bigg|_{p_z=p_0} + \frac{1}{2}\frac{\partial^2}{\partial p_z^2}\left(\frac{1}{m^*\overline{v}_z}\right)\bigg|_{p_z=p_0} (p_z - p_0)^2. \tag{35}$$

Physically this means that the helical pitch of the electron trajectory in magnetic field passes through an extremum. Then one can use a saddle-point approximation to calculate the integral in equation (32). As a result one gets

$$\overline{j}_\xi \propto \frac{1}{B^{3/2}} D\left(\frac{B}{\Delta B}\right), \qquad \Delta B = \frac{c\cos\chi}{ae}\left(\frac{\partial S}{\partial p_z}\right)_{p_z=p_0}. \tag{36}$$

Here D is an oscillating function of its argument with period 1; ΔB in equation (36) is determined by $p_z = p_0$. The exact form of the oscillation is determined by the form of electron trajectory on the ω'-surface at $p_z = p_0$.

3. Concluding remarks

In summary, we have worked out a theory of oscillation of the light-induced photocurrent in mesoscopic metal films as a function of an arbitrarily oriented magnetic field $\mathbf{B}$. We consider both the simplest example of spherical constant energy surfaces and closed constant energy surfaces of an arbitrary form. We have shown that for the second case the oscillations can be of two types. The first type can be called the limiting point oscillation as it is associated with a limiting point of the ω'-surface. It can exist for any type of constant energy surface, including spheres. The second type of oscillation can exist for more complicated types of ω'-surfaces. It is associated with the situation where the helical pitch passes through an extremum in a region of ω'-surface where the electrons have a substantial component of the velocity in the plane of the plate.

Indispensable for the observation of this phenomenon is the (partially) diffuse scattering of electrons from at least one surface of the sample. Under this condition the oscillation pattern can be observed in metals. In the case where there is a specular reflection from *one surface* of the film its effective width in the equation for the oscillation period should be doubled.

There are some essential differences between the oscillation of the surface current excited by light and the Sondheimer oscillation. The amplitude of the Sondheimer oscillation for the average conductivity $\overline{\sigma}_{\xi\xi}$ is proportional to B^{-4} for the first type and to $B^{-5/2}$ for the second type. The amplitudes for the oscillation of the photocurrent $\overline{j}_\xi$ are proportional to B^{-3} and to $B^{-3/2}$, respectively. The difference results from nonhomogeneous excitation of nonequilibrium electrons by the light. The tilt of the external magnetic field corresponds to an increase of the 'effective thickness' of the film in the oscillation period. This means an enhancement of the distance an electron has to pass in order to reach the opposite surface of the film.

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